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Continued :→ Topic — Double and Repeated limit, B.Sc III

State and Prove Moore-Osgood theorem,

Statement :→

Let the simultaneous $\lim_{\substack{x\to a\\ y\to b}} f(x,y)$ Exists and Equals to $l$ and

let the $\lim_{x\to a} f(x,y)$ exists for each constant value of $y$ in the nhd of $y=b$ and also

let the $\lim_{y\to b} f(x,y)$ Exists for each constant value of $x$ in the nhd of $x=a$

then $\lim_{x\to a}\lim_{y\to b} f(x,y) = \lim_{y\to b}\lim_{x\to a} f(x,y) = \lim_{\substack{x\to a\\ y\to b}} f(x,y) = l$

Proof :→

Since the $\lim_{x\to a} f(x,y)$ exists for each value of $y$ in the nhd of $y=b$.

we shall obtain an aggregate of these limiting values which define a function of $y$, say $F(a,y)$

thus we have $\lim_{x\to a} f(x,y) = F(a,y)$ —— (I)

Where $F(a,y)$ may or may not be identical with $f(a,y)$

Let $\varepsilon > 0$ be given

Since $\lim_{x\to a} f(x,y) = F(a,y)$, therefore there exists $\delta_1 > 0$ such that for each value of $y$ in the nhd of $y=b$

i.e $|y-b| < \delta_1$, we have

$|F(a,y) - f(x,y)| < \varepsilon/2$ —— (II)

for all $x$ satisfying $|x-a| < \delta_2$

Also From the Existence of Simultaneous limit at $(a,b)$, there Exists $\delta_2 > 0$ such that

$|f(x,y) - l| < \varepsilon/2$ —— (III)

for all $x, y$ satisfying

$|x-a| < \delta_2$, $|y-b| < \delta_2$

let $\delta = \min(\delta_1, \delta_2)$

the we have

$$|F(a,y) - l| = |F(a,y) - f(x,y) + f(x,y) - l|$$

$$\leq |F(a,y) - f(x,y)| + |f(x,y) - l|$$

$$< \varepsilon/2 + \varepsilon/2 \quad [\text{From (ii) and (iii)}]$$

it follows, therefore that $\lim_{y \to b} F(a,y) = L$

$\Rightarrow \lim_{y \to b} \lim_{x \to a} f(x,y) = L$ (From (1))

Similarly it can be shown that

$$\lim_{x \to a} \lim_{y \to b} f(x,y) = l$$

thus $\lim_{\substack{x \to a \\ y \to b}} f(x,y) = \lim_{x \to a} \lim_{y \to b} f(x,y) = \lim_{y \to b} \lim_{x \to a} f(x,y) = L$

[Remarks (i) Condition given in the theorem is only a sufficient but not a ~~necs~~ necessary] condition for the interchange of the order of Repeated limits

(ii) If the $\lim_{x \to a} \lim_{y \to b} f(x,y) = \lim_{y \to b} \lim_{x \to a} f(x,y)$

then $\lim_{\substack{x \to a \\ y \to b}} f(x,y)$ may or may not exist

End

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Probable question

1. Define Double and Repeated limits. Give Example.

2. Write a note on double limits and repeated limits of a function of two variables with special reference to their existence and Equality

3. State and Prove Moore-Osgood theorem

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